Inequalities (5) and (6), therefore, form a set of both necessary and sufficient conditions for the spin stabilized rocket to be stable,[5] in fact, asymptotically stable in the sense of Lyapunov.[6]

Condition (19) of Davis et al. may now easily be derived from the inequality

$$4(abd-b^2c-d^2)+(ab-2d)^2>0 \tag{7}$$

which reduces to

$$a^2-4\,c>0$$

on simplification even when $d=0$ holds true. Inequalities (5), (6), and (8) are essentially the stability conditions given by Nielsen and Synge.[7]

It may be mentioned that the method suggested can be applied when the coefficients in Eq. (4) are functions of some parameter, bounded, which is allowed to vary as in the case of motion with nonlinear aerodynamic forces and moments.

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Supporting Wire Interference Effects in Supersonic Near Wakes of Slender Bodies

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Introduction

HIGH-SPEED vehicles entering the atmosphere experience wake phenomena, which have an adverse effect on communications between the vehicle and some other transmitter and/or receiver located beyond the wake. Because of this communication problem, much investigation has been conducted to determine what exactly happens in the near and far wake.

One of the major problems encountered while investigating the near wake region of slender bodies, is that of supporting the model. Since the near wake region begins immediately behind the model and sometimes extends as far downstream as $X/H=4$, where X is the distance downstream of the base and H is the base height, the presence of any size sting will affect the base flow, the base pressure, and base heating. Side-mounted or wire-supported models are usually relied upon in order to obtain minimum disturbance measurements in the base region, with side-mounted systems being used primarily with two-dimensional models.

Due to the interference effects encountered when side-mounted systems are used for three-dimensional models, some investigators have turned to wire as a convenient method of model support. Considerable controversy has arisen in the past with regard to the amount of influence the wire supports have on the model flow patterns. The purpose of this paper is to investigate the effects support wires of different diameters have on the near viscous wakes of a wedge and a cone with the same included angle and base height or diameter.

Literature Review

The degree of interaction between the support wires and model flowfield, as investigated by previous authors,[1-6] has resulted in considerable disagreement. The majority of this disagreement is concerned with: how many body diameters downstream the wire effects are observed; whether the disturbances are confined to the plane of the wire; and whether the model's wake neck shifts position because of the presence of the support wires. Mirly and Selberg[3] indicated that in the viscous wake, the presence of support wires has no effect on the pitot pressure for ratios of wire to model base diameters equal to or less than 0.007. Schmidt and Cresci[5] found no induced disturbances beyond 250 wire diam, while Chapkis and Garnage[7] measured disturbance past 1500 wire diam downstream. Mirly and Selberg[3] reported that in the nonviscous flow region, the effect of pitot pressure was lowered because of an interference wire, and this effect was not confined to the plane of the wire. These findings were in agreement with Dayman,[1] who observed shock waves induced by the wire in and outside the plane of the wire, whereas Chapkis and Garnage,[7] along with Hromas,[2] reported that the effect of wire is felt mainly in the plane of the wire support and that useful data can be obtained out of the plane of the wires. Dayman[1] stated that for M(Mach Number) ≤ 2, the support wires had negligible effect on the wake, whereas for $M>2$ the wake neck moved towards the base of the body. Ragsdale and Darling[4] observed that only for large wire to base diameter ratios did the wake neck move towards the base. Mirly and Selberg[3] concluded that in the near wake, the support wire did not interfere with wake growth and shock wave position. Hromas[2] concluded that the wire support appeared to have a significant effect only on the pitot pressures and not on the static pressures, indicating that the wire support does not induce a system of shock waves in the plane of the wire, but rather creates a quasi-steady, rather complicated vortex pattern in the plane of the wire. Pierce and Beecham[8] have reported that for wires at large angles of attack there is a periodic shedding of the boundary layer, which results in vortices that shed alternately from either side of the wire. In view of these apparent disagreements about wire induced effects, it was felt that more data, were needed to resolve that exact effects of wire supports.

Test Apparatus and Experimental Techniques

Wind Tunnel

All experiments were conducted in the University of Missouri-Rolla supersonic axisymmetric wind tunnel, which is an enclosed, free-jet, intermittent flow facility with a Mach 3.15 nozzle, at a Reynold's number of 2.14×10^6 per in. The tests were conducted at an operating stagnation pressure of 140 psig and at an average stagnation temperature of 500°R.

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Wind-Tunnel Models

The first model tested in the near wake investigation was an aluminum wedge, 16° included angle, with a base height of 0.620 in. A 0.025 in. diam vertical hole was placed 0.503 in. forward of the trailing edge of the wedge and along its centerline to accommodate the interference wire. The wedge, when installed in the wind tunnel, became a two-dimensional model.

The support wire passing through the wedge was used for disturbance investigations only and was not needed for support purposes. The wire to be investigated was fastened rigidly to the top of the wind tunnel, out of the flow region, passed vertically through the wedge, and was inclined 50° forward and fastened to the bottom of the wind tunnel out of the flow region. This technique of having the wire vertical on the top surface and inclined on the bottom surface of the wedge, permitted investigation of the effects of varying the diameter of the interference and the effect of wire inclination.

The second part of the investigation was conducted with a wire-supported cone. The cone had a 0.620 in. base diam and a 16° included angle. The cone was machined from plastic and it had a length of 2.25 in. and a nose radius of 0.032 in. The forward and backward sets of two support wires were offset with respect to each other by 90° with the backward set rotated 45° with respect to the forward wires. The support wires passed completely through the cone, and were not rigidly attached to the model, which allowed the cone to align with the flow during test runs.

Pitot Pressure Probe

Pitot pressure results were obtained with a pitot probe that had an inside 0.010 in. diam. A small inside diameter pitot probe tip was used to insure that pitot measurements could be made inside the model boundary layer. The probe was mounted in a vertical transverse mechanism which allowed the probe to be moved up and down in the course of a run. The precise location of the pitot probe with respect to the wedge position was recorded simultaneously with the pressure on a two-channel recorder. The position signal was obtained as a changing voltage across a rectilinear potentiometer, actuated by the pitot transverse mechanism, which yielded a position accuracy of ± 0.002 in. Because of the small pitot tube size, a large time constant existed for a pressure reading to reach steady state, requiring discrete movement of the pitot probe system. Care was taken before each run, to insure that all pitot readings were made in the wire plane.

Schlieren Photographs

Schlieren photographs were taken of the wire-supported cone to determine the cone's wake neck location, rear stagnation point, and wake growth rate for different diameter support wires. Figure 1 is a Schlieren photograph of the cone model, while being supported with 0.007 in. diam wire.

Results

Wedge Investigation

The pitot probe was located 0.109 in. behind the wedge for all runs, $X/H = 0.176$. The wire diameter was varied from 0.007-0.025 in., which allowed the interference study at varying wire diameters downstream of the wire. The interference wires of 0.007, 0.015, 0.020, and 0.025 gave an equivalent downstream distance of 87, 41, 31, and 24 wire diameters, respectively. At the beginning of each pitot traverse the probe was located 0.140 in. above the wedge and moved in the course of the pitot traverse to 1.000 in. below the wedge.

A pitot traverse of the wedge without an interference wire installed was used as a base line comparison, in Fig. 2, against the pitot profiles of the wedge with the various diam support wires installed. The value for P_{t2}/P_{t1} in the reverse flow region, as described by Lewis and Behrens,[9] was found to be 0.083. The values of P_{t2}/P_{t1} were in good agreement with previous investigations made by Batt and Kubota,[10] Ohrenberger and Baum,[11] and Wu and Behrens.[12] The slope of the wake profile changed abruptly at $Y = \pm 0.375$ in., indicating that the probe was passing through the lip shock and from the viscous to the nonviscous region of the wake. This is in agreement with the Schlieren photographs which show that

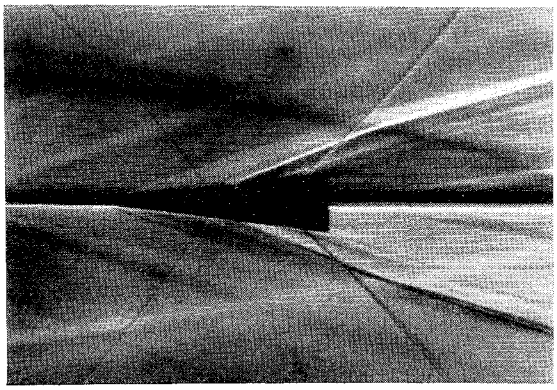

Fig. 1 Schlieren photograph of wire supported cone model.

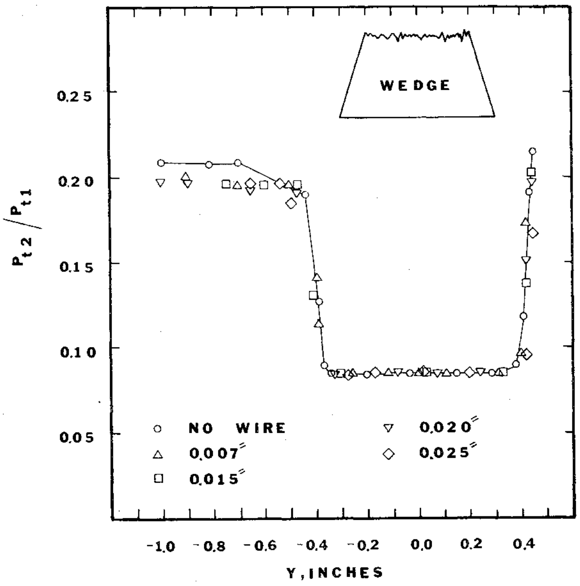

Fig. 2 Pitot pressure profile for wedge with various diameter interference wires.

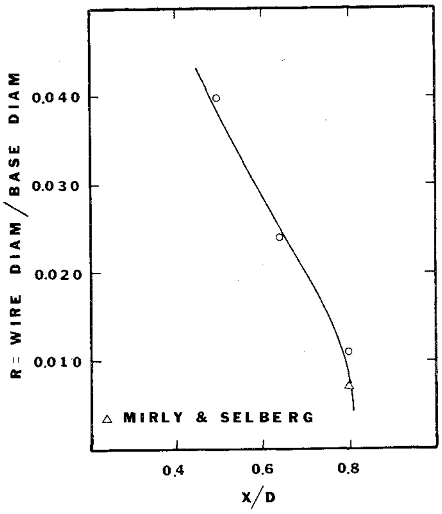

Fig. 3 Comparisons of wake neck locations.

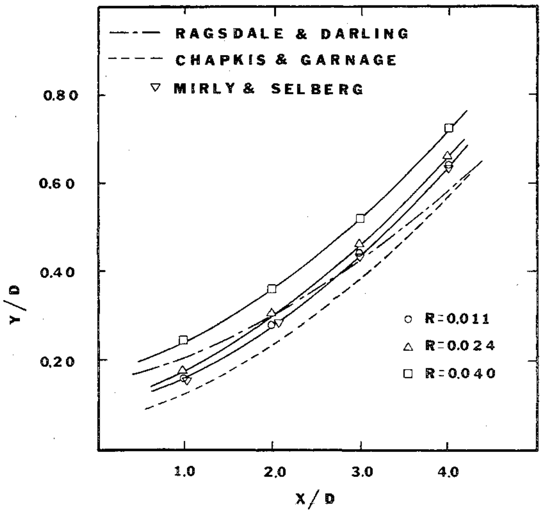

Fig. 4 Comparisons of wake shock locations.

the wedge lip shocks were located at $Y = \pm 0.370$ in. These photographs were used to determine the thickness of the boundary layer, which was found to be 0.113 in. This was larger than that predicted by the laminary calculations and indicates a transitional or turbulent boundary layer. The wake neck thickness was also scaled from the Schlieren photographs and found to be in agreement with measurement made by Batt and Kubota.[10]

Figure 2 shows negligible effects on the pitot pressure ratio for the various diameter interference wires investigated in the viscous or recirculation region of the wake. In the nonviscous flow region, Chapkis and Garnage[7] and Mirly and Selberg[3] noted lower pitot pressures in the plane of the support wire than found outside the wire plane. This is in agreement with the data shown in Fig. 2. This reduction in pitot pressure in the nonviscous wake is not dependent on the size of the interference wires, or the distance downstream of the wedge in equivalent wire diameters.

Cone Investigation

Schlieren photographs were made with the cone model supported by support wires of 0.007, 0.015, and 0.025 in. diam to investigate the effect that varying the ratios of wire to model base diameter R had on the model's supersonic wake.

A boundary-layer trip ring, 0.020 in. thick, was installed on the cone to determine if the cone had a turbulent boundary layer.[13] Schlieren photographs of the cone model with and without the trip ring, disclosed that the wake's width and location were the same for both cases. Thus the cone's surface was of sufficient roughness to cause a transitional or turbulent boundary layer.

Figure 3 is a comparison of the wake neck location for increasing values of R. The wake neck location was scaled from the Schlieren photographs. For the 0.007 support wire, $R/D = 0.013$, the rear stagnation point was estimated to be at approximately 0.8 base diam downstream of the cone base. Mirly and Selberg,[3] $R/D = 0.007$, found the rear stagnation point to be at approximately 0.8 base diam behind their cone. Ragsdale and Darling[4] found the rear stagnation point of a similar wire-supported cone to be at 0.9 base diam downstream and that as the value of R was increased, the rear stagnation point location shifted toward the model base. This finding is supported by Fig. 3, where a linear relation can be observed between the location of the rear stagnation point and the values of R; however, for values of R less than or equal to 0.011, the rear stagnation point location did not shift, but remained 0.8 base diam downstream. Chapkis and Garnage[7] working with a wire-supported cone, found the rear stagnation point to be about 0.8 base diam downstream from the cone base.

The wake shock width, scaled from Schlieren photographs, for the various diameter support wires is shown in Fig. 4. The wake shock locations shown for $R = 0.011$, 0.024, and 0.040 were in good agreement with Mirly and Selberg's,[3] Ragsdale and Darling's,[4] and Chapkis and Garnage's[7] results. Figure 4 shows that as R increases, the wake width was observed to increase, but the slope of the wakes remains constant. As the ratio of R was increased from 0.007-0.025 in., the wire-induced shock waves became more prominent in the cone's wake region.

Conclusions

Pitot pressure measurements behind a 16° wedge and Schlieren observations of an axisymmetric 16° cone at a Mach number of 3.15 with interference wires of varying diameters resulted in these conclusions: 1) In the viscous wake of the wedge, the presence of support wires of diameters 0.007, 0.015, 0.020, and 0.025 in., located both vertically and inclined with respect to the flow, had no effect on the pitot pressure ratio. 2) The pitot pressure ratio behind the wedge was found to be in good agreement with previous investigations made by Batt and Kubota,[10] Wu and Behrens,[12] and Ohrenberger and Baum.[11] 3) In the nonviscous flow region behind the wedge, the pitot pressure was found to decrease initially because of the presence of the support wire. However, enlarging the wire size resulted in no further reduction of pitot pressure. 4) The cone's rear stagnation point was found to shift toward the model base as the value of R (ratio of wire diameter to base diameter) was increased from 0.011-0.040. 5) For values of R less than or equal to 0.011, the presence of the support wires did not affect the location of the cone's rear stagnation point, i.e., it remained at a constant value 0.8 base diam downstream of the cone base. 6) Increasing the value of R resulted in a slight increase in the width of the cone's near wake; however, the slope of the wake remained constant. 7) For values of R ranging from 0.011-0.040, the wake growth for the wire-supported cone was found to be in good agreement with the results of previous investigators.

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Synchronous Satellite at Less Than Synchronous Altitude

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I. Introduction

THE establishment of a synchronous (stationary) satellite at less than synchronous altitude was first examined in 1967[1] where it was found that a high-strength tapered cable (boron fiber with a 0.5×10^6 psi yield stress) could support a payload at near half synchronous altitude. The results of that study showed that the mass of the cable plus counterweight (above synchronous altitude) would be about 80 times that of the payload. It was recognized, however, that the problems of deployment and stabilization could be serious.

Potential uses of a synchronous satellite at less than synchronous altitude may include communication, navigation, and surveillance missions. If, for example, an omni or broad beam antenna is utilized then the transmitted power (and weight) is only a fourth that at the synchronous altitude (inverse square law). Or, if the same power is available then a fourfold increase in the amount of information transmitted is possible. Other potential benefits may include a signal to noise ratio increase for Earth observation missions and a linear sensor resolution improvement with the decrease in altitude. Night time illumination of selected Earth areas may also be feasible with the intensity of illumination being proportional to the mirror diameter and inversely proportional to the orbital altitude. For example, an 80-m diam mirror which is flat to within a small fraction of the sun's angular diameter and which is located at one-half the distance from the Earth to the synchronous altitude could provide full moon illumination ($\sim 10^{-2}$ lm/ft^2) over a 100-square-mile area.

This note extends the results of Ref. 1 and examines the possibility of using a viscoelastic organic material Kevlar which has nearly twice the strength-to-weight ratio of boron steel fibers. It will be shown that a significant reduction in the total satellite weight can be achieved. Also considered will be possible deployment and stabilization methods.

II. Cable Equlibrium Equations

The schematic diagram of the cable-connected satellite system in equlibrium is shown in Fig. 1. Using the notation of Ref. 1, the derivative of cable tension T with respect to the radial coordinate r is

$$\frac{dT}{dr} = \rho A \left(g a^2 / r^2 - \omega^2 r \right) \tag{1}$$

where ρ = mass density of cable, g = gravitational constant at the Earth's surface, ω = orbital angular velocity, $A = T/\sigma$ = cross-sectional area of cable, σ = cable design stress, and a = Earth's radius.

Integration of Eq. (1) yields

$$T/T_m = \exp - \gamma (1-s)^2 (1/2 + 1/s) \tag{2}$$

where $\gamma = \rho (g a^2 \omega)^{2/3} / \sigma$ and $s = r/\lambda$. The cable mass M_c and the counterweight mass M_2 can be given as (Ref. 1)

$$\frac{M_c}{M_1} = \gamma \left[\frac{1}{s_1^2} - s_1 \right] e^{\gamma (1-s_1)^2 (1/2 + 1/s_1)} \times \int_{s_1}^{s_2} e^{-\gamma (1-s)^2 (1/2+1/s)} ds \tag{3}$$

$$\frac{M_2}{M_1} = - \frac{(s_1^{-2} - s_1) \exp \gamma (1-s_1)^2 (1/2 + 1/s_1)}{(s_2^{-2} - s_1) \exp \gamma (1-s_2)^2 (1/2 + 1/s_2)} \tag{4}$$

Here

$$s_1 = r_1/\lambda, \; s_2 = r_2/\lambda$$

At

$$r = r_1, \; T_1 = M_1 (g a^2 / r_1^2 - r_1 \omega^2)$$

III. Numerical Evaluation

Equations (3) and (4) were integrated numerically and plotted in Fig. 2 for boron steel ($\gamma = 6.56$) and Kevlar ($\gamma = 3.81$) as a function of radius s_2. It is apparent that the lowest overall mass ratio is obtained at $s_2 = 1.4$ and that a considerable reduction results from the use of Kevlar. Similar results can be observed from Fig. 3 which is a plot of Eq. (4). The crossing of curves indicates that the heavier material (steel) is more effective for longer cables above ths synchronous altitude. These results show, for example, that for a Kevlar ($\gamma = 3.81$) cable extending from 10,000-n. mi altitude to about 28,000 n. mi (18,000 n. mi long) the cable plus counterweight $(M_2 + M_c)/M_1$ ratio is 25 and $M_2/M_1 = 5$ which implies considerably lower total satellite weight than would be required for the boron steel material. The tensile strength of resin impregnated Kevlar strands is 525,000 psi and its density is 1.45 g/cm^3 compared to 500,000 psi and 2.4 g/cm^3 for the boron fiber. The strength to weight advantage of Kevlar is thus apparent although its somewhat increased sensitivity to uv radiation may require the development of a protective coating or its replacement by a graphite or fiber glass material with similar properties.

IV. Deployment and Stabilization Considerations

The deployment of a gravitationally stabilized 10,000-20,000-n. mi cable in orbit is likely to be a difficult process at best. Several studies, for example, have considered the deployment of cables 1-3 n. mi long. The approach of Ref. 2 was a two-stage deployment process consisting of a spring imparted separation velocity to the two end masses which are separated to a preset length of cable. Capture by the gravitational gradient occurs and the librational motion is then removed without dissipative damping by a "deadbeat" process of inertia adjustment initiated at a predetermined time. This begins during the first swing toward the vertical with the release of a second preset length of slack tether. The end bodies again move apart in separate coplanar orbits until the tether (cable) runs out of slack and the line joining the bodies approaches the local vertical at near zero angular rate. Tension-limited snubbing with modest energy absorption is required for this method of deployment.

The deployment of the present cable-connected satellite and its stabilization relative to the local vertical in the synchronous

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